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控制太阳活动的混沌吸引子

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摘 要

随着非线性科学研究的进展，可利用表征太阳活动的太阳活动指数组成的时间序列来寻找或许存在的太阳混沌吸引子，并计算其关联维数、最大 Lyapunov 指数以及其它特征量。文中综述了用非线性科学的某些概念来研究太阳活动的进展及其在太阳活动预报方面的一些应用。

关键词 混沌现象 — 方法：数据分析 — 太阳：活动 — 太阳黑子

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The Chaotic Attractor Governing Solar Activity

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Abstract

Following the research progress in nonlinear science, many people have used the time series of presently available solar activity indices characterizing solar activity to search for the suspected solar chaotic attractor and to calculate the correlation dimension, the largest Lyapunov exponent and the characteristic parameter of its other behavior. The progress in the study of solar activity and some applications to the prediction of solar activity using the idea of nonlinear science is presented in this review.

Key words chaotic phenomena—method: data analysis—Sun: activity—sunspot

1 Introduction

Solar activity is caused by the magnetic activities, which can be measured by means of sunspot relative number. The sunspot number becomes a solar index characterizing the overall

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level of solar activity. It has been used since Wolf proposed it in 1848, and the sunspot records can extend back to 1749 or even earlier. Reliable and complete records have been accumulated about one and half centuries[1].

The evolution of these data with time showed a violent and irregular variation. Statistical analysis of the sunspot number showed that its frequency and amplitudes of the spectral peaks varied with the data length. The long-term predictions of the sunspot number made in this purely numerical statistical method have not fared well[2,3]. The nonlinear predictions of the sunspot number in practice also showed that the predictive accuracy fell off with increasing predictive time interval[4]. Statistical analyzes of other indices for solar activity indicated[5] that the most stable periods in apparent periods were 11 years (Wolf-cycle), 22 years (Hale-cycle), 88 years (Gleissberg-cycle) and 352 years (Elatina-cycle), which can be arranged in period-doubling. They can be expressed respectively as: 11×2^0 years, 11×2^1 years, 11×2^3 years, 11×2^5 years. The existence of prolonged minima of solar activity in history known as: Oort (1010–1050), Wolf (1280–1340), Sporer (1420–1530), Maunder (1645–1715) and Dalton (1800–1820) has been shown also by means of analyzing the time series of variations of radiocarbon 14 concentration in tree rings. Solar activity appeared transient intermittence in these minima. The research demonstrated[18] that some nonlinear characteristic of solar activity can be recurred by the nonlinear solar-dynamo model. These facts demonstrate that the evolution of solar activity has an exceedingly high complexity and maybe exists chaotic movement.

Research also indicated[6] that any macroscopic real system was a dissipative system whose volume in phase space would continuously contract in its evolution process, and an evolution with various initial conditions would tend to a same result or few different results. Such limit sets are called attractor. Following volume contracts, all trajectories tend to an attractor which has lower dimension than that of original phase space. Attractor can be divided into fixed attractor, periodic attractor, quasiperiodic attractor and strange (chaotic) attractor, and the dimension number of chaotic attractor is fractional dimension in general. Attractor depicts the whole state of dynamical system, so, investigating attractor can make us understand the long-term evolutionary behavior of the dynamical system.

Following the research progress in nonlinear science, the fractal theory has become a branch of sciences in the research of regularity of complicated phenomena in nature and human society. Exploiting fractal, we can study the infinitely embedding self-similar structure which the chaotic movement has, and discover the systematic complexity, moreover explore its dynamic mechanism. In 1980, Packard *et al.* [7] provided a technique reconstructing a phase space by means of time series. Takens[8] presented the embedding theorem of time delay coordinate method in mathematics, making multiple dimensional information hiding in one-dimensional time series displayed, and reconstructing the attractor, which is used to investigate geometric characteristic of the time series. In 1983, Grassberger and Procaccia[9,10] used a method of extended time series and presented a technique for determining the correlation dimension of attractor characterizing dynamical system by correlation function. The correlation dimension is an optimal approximation of the fractal dimension in a chaotic dynamical system. This is the famous G-P technique in nonlinear science. Because the fractal dimension is one of the basic characteristic parameters with exceeding deep physical significance in long-term evolution of a dynamical system, it can be used to judge whether there is existing strange attractor or not in system, and whether there is existing intrinsic randomness or not. Its size also provides the extent of complexity of the investigated system, i.e., the minimum number needed for independent variates describing the system evolution, in addition, it has intrinsic connection with Lyapunov exponents also, and can provide predictable time scale of the system.

In 1985, Wolf *et al.*[27] presented a mathematical method for calculating the largest Lyapunov exponent of time series from experiments. The largest Lyapunov exponent is a most important parameter to determine whether there is chaotic behavior or not in the system. It measures the average convergence or divergence rate of neighboring trajectories in phase space. Its sign can be used to determine the long-term evolutionary behavior of the system. If the exponent is above zero, the system will be a deterministic chaotic system, the initial neighboring trajectories will be divergent, and the system will be sensitive to initial condition. So its long-term deterministic behavior will be unpredictable, even if the deterministic equation sets characterizing the system are known. Its size of numerical value shows the average loss rate of information of initial condition. Conversely, if the largest Lyapunov exponent is under zero, the system will not be sensitive to initial condition, and its long-term deterministic behavior will be predictable. If it equals zero, the system will correspond to stable border.

The progress in nonlinear science has attracted great attention in many subjects including solar physics. From then on, the articles, in which one-dimensional time series characterizing solar activity, such as a variety of solar index, are exploited to search out the possible existent chaotic attractor governing solar activity and its characteristics have been successively given.

2 Research Progress

Early in 1981, on the basis of the fact that nonlinear ordinary differential equations can show stochastic behavior within a certain parametric range, Ruzmaikin[11] first proposed a nonlinear solar dynamo model with strange attractor, and introduced the concept of strange attractor into the research on solar cycles and explained the stochastic behavior in the larger time scale of solar cycles. Following the appearance of the G-P technique, many people exploited the time series of the sunspot number to search out the possible existent solar chaotic attractor. Spiegel *et al.* [12] obtained the fractal dimension about 5-dimension using the daily sunspot number during 100 years. Ostryakov *et al.* [13] calculated the fractal dimension for the monthly mean sunspot number over three different period of time and found the fractal dimension of 4.3 for 1749–1771, 3.0 for 1792–1828, and 4.0 for 1848–1859, respectively, but their data used are not reliable in early sunspot records and the number of data is too poor, as the longest series have only 432. They also calculated K_2 entropy and the largest Lyapunov exponent for the monthly mean sunspot number for 1749–1810 and obtained a conclusion about the existence of low-dimensional chaotic attractor and its predictable time scale of under 5 years. The fractal dimension of 6.3 for the 9-day mean sunspot number from 1932 to 1982 is obtained by Shen Mei *et al.* [14]. Gizzatullina *et al.*[15] used the time series of radiocarbon 14 data in annual rings of ancient tree from 4300 BC to 1950 AD to look for the correlation dimension of the solar attractor and K_2 entropy, and the result was 3.3 and 0.6/year, respectively. Mundt *et al.* [16] used the monthly mean sunspot number from 1749 to 1990, and obtained the correlation dimension, the largest Lyapunov exponent and the predictable time scale of the system by means of low-pass filtering, and the result was 2.3, 0.02/month, and 4 years, respectively.

But, Price *et al.* [17] used the G-P technique to analyze the monthly mean sunspot number and the monthly smooth sunspot number from 1749 to the present, and stated that there was no conclusive evidence for low-dimensional chaos in any of the studied time series and that the scaling regions found by other authors can be spuriously produced by the use of filtered data sets. Carbonell *et al.* [18] thought that the calculations made before were either too poor in the number of data, or too short in the exploited delay time, so they analyzed the different time series made up of the daily sunspot number from 1818 to 1990 and the daily sunspot area from 1874 to 1989 using the same technique. Their results did not show any evidence for the presence of low-dimensional deterministic chaos in these time series, because the correlation dimension never reached saturation with increasing the embedding dimension in the calculations. They thought that to explore the chaotic behavior of solar cycles by means of the G-P technique would need longer and reliable data sets, covering the periods of reduced activity.

There have been many researches on the minimum size of noised data required for accurate determination of the attractor dimension using the G-P technique, but no consensus has been reached. Well-known numerical experiments on the Lorentz system, the Henon mapping etc. indicated that over 500 data points would suffice, while for data sets affected by larger noise, more data points seem to be required. Smith[19] suggested that the G-P technique required a minimum data size: $N_{\min} \geq 42^M$, M being the largest integer less than the fractal dimension D. For example, when $M = 2$, $N_{\min} \geq 1764$; when $M = 3$, $N_{\min} \geq 74088$. And we can at most calculate attractor within 5 or 6 dimensions. Ruelle[20] suggested a criterion for test of the calculating results: for the delay time $\tau \geq 10$, we should have $D \leq 2 \cdot lgN'$, N' being the sample size when τ is taken as sampling time interval.

We[21] calculated the correlation dimension of the monthly mean variation of the Ottawa 10.7cm radio flux and found a correlation dimension of 3.1. In our previous papers[22,23], we strictly analyzed and discussed the influence of the reliability of the data and the size of the data on the calculated results, and calculated the correlation dimension D of the monthly mean sunspot number in modern era solar cycles (January 1850–May 1992). We found that $D = 2.8 \pm 0.1$, its largest Lyapunov exponent $\lambda_1 = 0.023 \pm 0.004$ bits/month, and the upper limit of the theoretical time scale for deterministic prediction, $t = 3.6 \pm 0.6$ yr. The results indicated that the evolution for the monthly mean sunspot number was neither periodic nor quasiperiodic, but was chaotic, and the evolution of its trajectory in phase space was a low-dimensional chaotic attractor, which could be described by at least three and at most seven variates. Ruelle's criterion is satisfied in our calculation, and Smith's requirement is nearly satisfied in our case. We[24] also estimated the time scale of a transition from a high-dimensional chaos or stochastic behavior at shorter time to a low-dimensional chaotic behavior at long time using the G-P technique, and found the transitional time scale about 8 years. This indicates that the low-dimensional chaotic behavior of the attractor governing solar activity operates completely at time scale longer than about 8 years and a high-dimensional or stochastic process operates at time scale shorter than about 8 years. We[25] also discussed several noteworthy problems in using the G-P technique to search for chaotic behavior of solar activity. These problems are data reliability, data length, filtering problem, and choice of time delay and optimum scaling range. As long as these problems are strictly considered, the existing low-dimensional chaotic behavior for the monthly mean sunspot number in modern era solar cycles (1850–the present) can be determined by means of the G-P technique, and the largest Lyapunov exponent can also be calculated using the method proposed by Wolf *et al.* [27]. But in the previous analyzes and calculations some authors used unreliable data including that before 1849 or that of a different statistic population; or used the low-pass filtering to increase the signal to noise ratio, whereas the filtering would result in systematic errors in determining attractor dimension and would introduce additional Lyapunov exponents[26], and that would induce larger phase shifts and change the structure of the data series; or used shorter data sets; or used the uncertain scaling range; or used too small time delay. All of these resulted in the uncertainty of the calculated results. Recently a paper[28] indicated that the sampling time interval of the system was also an important parameter, which has influence on the calculated results.

We [23] also discussed the stability of the largest Lyapunov exponent estimate λ_1 for the monthly mean sunspot number with increasing data length. As long as the data size was above 1300, the estimate of λ_1 asymptotically tended to 0.023 ± 0.004 bits/month with increasing data length with a little fluctuation. Certainly, to get a reliable estimate, enough long data are required. Wolf *et al.* [27] pointed out that the number of data points required to adequately represent an attractor increases with increasing dimension of attractor. The data size should be about 10^D–30^D, D being the dimension of attractor.

It needs a large amount of data to determine the chaotic behavior of the system using the G-P technique. However, in general case, the data size obtained by means of an experimental observation is always limited. In 1990, Sugihara *et al.* [29] proposed a nonlinear predictive approach based on the method of distinguishing chaos from measurement errors in time series, in order to estimate the number of dimension and the largest Lyapunov exponent of the system, and to determine whether there is chaotic behavior in the system. The approach does not need a large amount of data as compared with the G-P technique. Investigation[29] also indicated that, for a chaotic time series, the accuracy of nonlinear prediction falls off with increasing predictive time interval, whereas for uncorrelated noise, the predictive accuracy was roughly independent of the predictive time interval in general. Tsonis *et al.* [30] further pointed out that, for short predictive time and chaotic systems, the $log[1-\rho(t)]$ would be a linear function of predictive time step t, and for the stochastic model of fractional brownian motions, it would be a nonlinear function of predictive time step t, where $\rho(t)$ is the correlation coefficient between predicted and actual observing values. Rozelot[3] applied the approach to the time series made up of the yearly mean sunspot number from 1749 to 1992 to calculate the correlation coefficients of predictive values in back half section, and obtained also the results as follows: the fractal dimension in the system was around 3, the largest Lyapunov exponent was 0.05/year and the upper limit of the theoretical time scale using the sunspot number to make deterministic prediction was 2–4 years. It also declares that solar activity is governed by a low-dimensional chaotic attractor and solar activity has a chaotic characteristic. These results agree with the results obtained by us.

From the investigation mentioned above, according to the theorem about Lyapunov exponents given by Haken[31] and the relationship[32] between the fractal dimension and the Lyapunov exponents, by now, we have already obtained the characteristics of the chaotic attractor governing solar activity as follows. Its fractal dimension is 2.8. It can be described by means of three Lyapunov exponents in 3-dimensional phase space: $\lambda_1 = 0.023$ bits/month, $\lambda_2 = 0.00$ bits/month and $\lambda_3 = -0.029$ bits/month. This indicates that, being averaged along the phase trajectory, λ_1 describes its instability, and the neighboring trajectories are divergent, whereas λ_3 describes the convergence to attracting manifold, its volume is steady in the evolutionary procession on the whole, and λ_1 is below $|\lambda_3|$. This suggests that this chaotic system is a strong dissipative and faint chaotic system. The low-dimensional chaotic behavior of the chaotic attractor is displayed primarily on the time scale only above 8 years or so, but on the time scale below 8 years or so, what is displayed is a high-dimensional chaos or intrinsic stochastic behavior. As a whole, it only needs 3–7 variates to describe the system, and the deterministic predictable time scale is 3.6 years.

3 Applications

The concept of nonlinear science and the structure of the low-dimensional chaotic attractor governing solar activity is applied to the field of investigation and prediction of solar activity, and there are a lot of significant applications not only in theory, but also in practice. For instance:

(1) The noisy background of the monthly mean sunspot number can be determined by means of the fractal concept. Hao Beilin[33] pointed out that while the time series was used to determine the scaling range and to estimate the correlation dimension, we could also determine the size of noisy background of experimental data series. The noisy background equals approximately to the distance r of the slope of D_d changing to embedding dimension d. So, we can get that the noisy background of the monthly mean sunspot number is about ± 6 in average, and it corresponds to a noisy level of 10 percent.

(2) McIntosh[1] pointed out that attempts to predict the exact time of the sunspot maximum

may be futile in the prediction of sunspot cycles. We[23] explained this problem using some concept of nonlinear science. Because of the sensitivity of the chaotic system to initial condition, there is $\delta x(t) = \delta x(0)\exp(\lambda_1 t)$, where $\delta x(t)$ and $\delta x(0)$ are the errors at time t and at $t = 0$, respectively. According to the error theory, $\Delta t \approx \Delta\lambda_1/\lambda_1^2$, where $\lambda_1 = 0.023$ bits/month for the monthly mean sunspot number, which is far less than 1, therefore, Δt will have the larger fluctuation as long as λ_1 changes somewhat. For example, if λ_1 has a change in a value of 0.001, then Δt would have a fluctuation of about 2 months. We notice from the estimate of λ_1 that λ_1 certainly has some fluctuation. The largest fluctuation is about 0.004 bits/month, and Δt will have the largest fluctuation of 8 months. Therefore it is very difficult to predict the exact evolutionary time, because there are some measuring errors in the sunspot number by an observer, and it is also influenced by the intrinsic stochastic behavior of the sunspot number itself. Therefore, the observing records of the sunspot number limit us to make an exact prediction on time.

(3) Layden *et al.*[2] pointed out that there are currently two broad approaches to predict solar cycles. One is purely numerical statistical approach in nature. Other approach is a precursor technique. But the predictions made in the former have not fared well in practice. Because the approach is based on the search of multiple periods and amplitudes capable of reproducing the past cycles, while the predicting relies on the assumption that the phenomenon is really periodic, and that the important periodicities have already shown up in the data. But, as recently known[23], according to the estimate of λ_1, the long-term evolution of the monthly mean sunspot number is not strict periodic, nor quasiperiodic, but chaotic. So this approach can not be used to the long-term prediction and can only be used to the short-term prediction, and the predictive time scale is solely $t = 1/\lambda_1 = 3.6$ years. Therefore the former is useless to predict exactly the long-term evolutionary behavior of solar cycles. In the practice of predicting maximum of solar cycles, it has been found that the purely numerical statistic approach can not be used until a solar cycle actually begins and requires a year or more of data on the rising phase[1]. This has been proved in the practice of the prediction of 22-cycle.

(4) In predicting solar activity using the neural network technique, for example, the yearly mean sunspot number[34], it is required to estimate the least number of independent variates restructuring the dynamics of solar activity, the unit number of the neural network input-layer and other parameters. The structure of solar chaotic attractor can be used to provide the optimal values of these parameters, thus we can determine the structure of the neural network using these structure parameters of solar attractor and obtain the better predictions of solar activity using the neural network technique at last.

Mundt *et al.*[18] used the nonlinear predictive method proposed by Farmer *et al.*[37]. We[4] used a method known as inverse problem in the chaotic time series proposed by Casdagli[38] to predict the monthly smooth sunspot number. Two methods need first to determine the dimension number of the system, to choose the optimal delay time, to reconstruct proper phase space, to choose the optimal neighboring region of phase trajectory nearby predictive point, even to use the largest Lyapunov exponent to improve or compensate predicted values of longer time steps in advance, then we can get better predictive results.

(5) Research[4] showed that, to make the prediction of several time steps in advance, it was essential to consider the chaotic behavior of the system in order to resolve the problem about the drop in the nonlinear predictive accuracy with increasing prediction time step in the dynamic system characterized by means of the chaotic time series. To obtain improved predictions, the estimate of the largest Lyapunov exponent of the system and the known predictive error of certain time step, obtained by comparing the predicted with the observed value, is taken respectively as an approximation of the exponential rate and the initial error of the predictive model and the calculated divergences of trajectories in phase space with time are added to original predictions. Therefore this technique compensates or improves the drop in the nonlinear predictive accuracy with increasing predictive time interval, caused by the intrinsic stochastic behavior (chaos)[35]. The practice indicated that this technique was feasible and improved greatly the prediction of the smoothed monthly sunspot number, and was superior in predictive accuracy to the other one used at present. In the prediction of the smoothed monthly sunspot number for the maximum epoch, the improvement was even more evident.

(6) According to the general dynamic characteristic of a chaotic system, the maximal values of solar cycles were made up a series by Kremliovsky[36]. He used the one-dimensional linear mapping with chaotic intermittent behavior to simplify actual dynamics, and got an approximate model about the evolution of the maximum for solar cycles. From this model, as long as a maximum of the solar cycle is known, the maximum of the next solar cycle can be predicted. For example, if the maximum range from 146 to 166 for 22-cycle is known, the predictive maximum for 23-cycle would be 160 ± 15. Using this mapping, the long-term evolution of solar activity can also be further researched and understood.

From the above-mentioned summaries of the application of analyzing and predicting solar activity, particularly in the predictive models of medium-term and long-term solar activity, the low-dimensional intrinsic stochastic behavior (chaos) of solar activity must be considered. These applications obviously can be introduced into other systems made up of chaotic time series. But, we should note, though the theory of fractal and chaos have rapid progress and have been applied to many fields, some problems still remain to be further investigated and resolved.

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